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DISTRIBUTION OF IMPACT MOMENTA OVER A SURFACE TREATED WITH A WATER-DROP JET

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A model of a random field of the momenta of impact forces acting on a plane surface treated with a water-drop jet has been developed. Based on the calculation and experimental data, it is shown that the action of individual drops is statistically independent in time and coordinates.

Treatment of a surface with a water-drop jet has found practical use in various fields of technology [1–3]. At the same time, almost without exception the control of the process up till now has been based on empirical dependences even though elementary acts of the action of individual drops are fairly well understood. There are approximate solutions of this problem, based on the regularities of the distribution of shock waves in a striking drop (see, for example, [4–6]), and solutions obtained with the use of modern computational means and giving a more exact and complete idea of the mechanism of interaction of a drop with a surface and the forces arising in this case [7–10]. The action of a water-drop jet considered as a stochastic ensemble of drops is much less understood. The available integral characteristics of the action of a water-drop jet are inadequate to prognosticate the damage parameters and control the erosion process.

Experimental distributions of the dimensions and velocities of drops have been obtained in [3, 11]. Based on them, the geometric characteristics of the wakes of a water-drop jet on a distant coating have been investigated [12].

The aim of the present work is analysis of the statistical characteristics of a random field of the impact momenta acting on a surface treated with a water-drop jet.

Model. A scheme of treatment of a surface with a water-drop jet is shown in Fig. 1. Among the main parameters of the process, which should be determined and measured, are the water pressure at the input to the nozzle p, the diameter of the nozzle d_0, the distance from the nozzle to the treated surface L, the velocity of travel of the nozzle relative to the surface v, the diameter D in the case of a jet with a round cross section, or other dimensions, for example, the width B and thickness h in the case of a so-called plane jet with a rectangular cross section [3, 11]. The following parameters are typical of the processes of treatment with a water-drop jet: p = 50–200 MPa and d_0 = 0.2–0.5 mm [1–3]. The parameters D, B, and h depend on the pressure p, the distance L, and the diameter and design of the nozzle [1–3]; however, in the present work, this dependence as well as the kinetics of breakdown of a jet into drops were not investigated.

It is assumed that, at any distance from the nozzle, the jet breaks down into drops forming a stochastic system and that the velocities of the drops at the instant they strike the surface are determined by the initial velocity of the jet at the output of the nozzle and the resistance to their movement, depending, in turn, on their dimensions and shape.

According to the experimental data [3, 11], the distribution of the diameters of the drops in a water-drop jet can be described using the Weibull distribution known in the theory of liquid atomization as the Rosin–Rammler distribution. For the velocities of water flowing from the nozzle and the distances from it considered in the present work, these distributions for round and plane jets are similar in the values of the parameters. Because of this, the calculations

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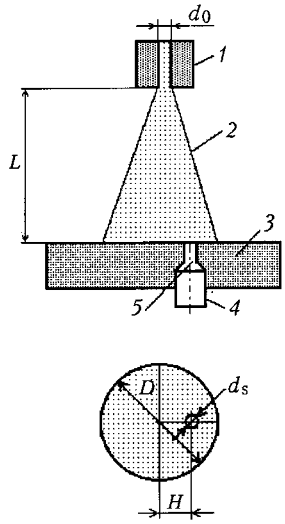


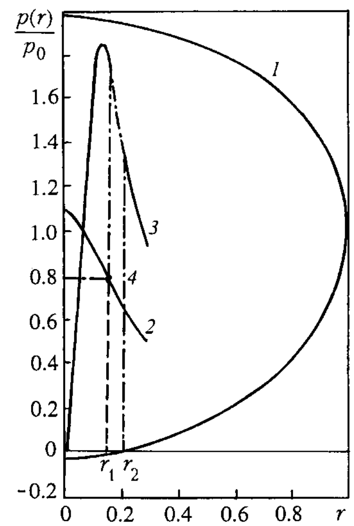


Fig. 1. Scheme of treatment of a surface with a water-drop jet: 1) nozzle, 2) jet, 3) sample, 4) detector of acoustic emission, 5) sensing element of the detector.

Fig. 2. Shape of a drop (1) and pressure distribution (2–4): 2, 3) distributions calculated by formulas (2), 4) resulting profile.

were performed on the assumptions that the mean diameter of the drops is equal to 0.06 mm, the variation coefficient is 1.1, and the maximum diameter of the drops is limited to the diameter of the output hole of the nozzle.

The ith drop of a diameter d_i is formed within the time $\tau_i = V_i/Q_i$, where $V_i = \pi d_i^3/6$ is the volume of the drop and Q is the rate of water flow through the nozzle, depending on the nozzle diameter and the water pressure. If drops of different diameters are formed in a random sequence at the instant the jet flows from the nozzle, the time intervals within which drops are formed as well as the drop diameters have a distribution similar to the Weibull distribution, but their variation coefficients are approximately two times larger.

The drops are retarded in the region of their movement to the treated surface because of the air resistance. The dependence of the velocities of the drops on their diameter at the instant they strike the surface can be taken as follows [12]:

$$u = k_u \sqrt{2p/\rho_w} \exp\left[-\frac{3c_z\rho_a L}{4\rho_w d}\right]. \tag{1}$$

The water pressure at the input of the nozzle is usually pulsating. The pressure pulsations also influence the velocities of the drops [12]. The distribution of the drop velocities, following from relation (1), agree with the experimental data of [11] if $c_z = 0.15$. This means that real drops are not spherical.

According to the known theoretical and experimental data [3–10], in the central part of the region where a drop contacts with the surface ($r < r_1$) (Fig. 2), the pressure is distributed practically uniformly and is related to the so-called Zhukovskii impact pressure $p_w = \rho_w u c_w$. At the periphery of the contact region ($r_1 < r < r_2$), the pressure has a peak component. To simplify the calculations, the pressure arising on the surface as a result of the impact of an individual drop was calculated by approximate formulas derived based on the analysis of the distribution of shock waves in a drop [6]:

$$p_1(r) = A \exp\left\{[\gamma u f_1(r)/c_w \sqrt{1 + f_1(r)}] - 1\right\},$$

$$p_2(r) = A \exp\left\{[\gamma u f_2(r)/c_w (1 + f_2(r)] - 1\right\}, \tag{2}$$

where A = 321 MPa, γ = 7,

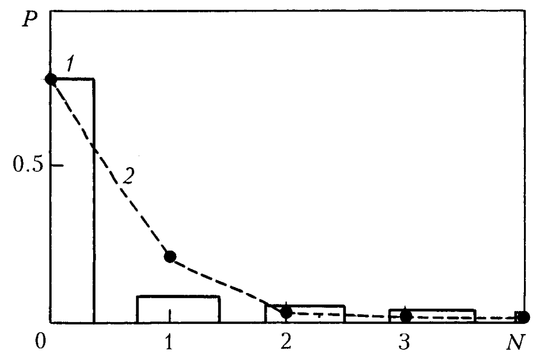


Fig. 3. Distribution of the number of drops acting simultaneously on the surface of the sensing element accoring to the stochastic model of a jet (1) and the Poisson law with parameter n_c (2).

$$f_1(r) = \frac{u\sqrt{R^2 - r^2}}{\sqrt{(c_w - u^2)\,r^2 + u^2R^2}}\,; \quad f_2(r) = \frac{c_w u r^2 \sqrt{R^2 - r^2}}{(c_w - u^2)\,r^2 + u^2R^2}\,.$$

The radii of the regions r_1 and r_2 related to the radius of the drop are determined as functions of the time of drop spreading t:

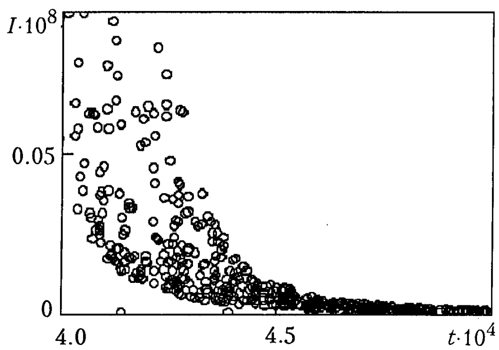
$$r_1 = R^{-1} c_w t\,, \quad r_2 = R^{-1}\sqrt{2Rut - (ut)^2}\,.$$

Within the time interval t_c after the contact of the drop with the surface, the ring peak-pressure region disappears; in this case, $r_1 = r_2$. The calculations have shown that the pressure peak on the contour is practically absent even at a contour-area radius of $\cong 0.5R$ (see Fig. 2). Consequently, the effective action of the drop pressure on the surface is limited by the time t_c within which the peak region exists at the periphery of the region of contact of the drop with the surface. Because of this, the quantity t_c can be considered as the time of action of an impact momentum on the surface. The distribution of this quantity is related to the distribution of the dimensions and velocities of the drops. The mean quantity $\langle t_c \rangle$ for the above-indicated values of the nozzle diameter and the water pressure is an order of magnitude smaller than the average time of drop formation $\langle \tau \rangle$. For example, $\langle t_c \rangle / \langle \tau \rangle = 0.26$ at a water pressure of 100 MPa and a nozzle diameter of 0.2 mm. This ratio estimates the number of drops ν_c acting simultaneously on the surface. According to the calculations, for a stochastic ensemble of drops, whose diameters are distributed by the Weibull law with the above-indicated parameters, the distribution of the quantity ν_c adheres to the Poisson law with the parameter $n_c = \langle \nu_c \rangle \cong \langle t_c \rangle / \langle \tau \rangle$ (Fig. 3). With increase in the rate of water flow through the nozzle (with increase in the water pressure and in the nozzle diameter), $\langle \tau \rangle$ decreases and the average number of drops simultaneously striking the surface increases; however, the probability of simultaneous action of more than two drops remains small. Therefore, it may be suggested that drops act independently under typical conditions of the surface treatment.

The kinetic momentum of a drop is $K_i = \rho_w u_i V_i$ at the instant it touches the surface. The forces acting on the surface as a result of the impact of the drop were calculated by integrating the pressure given by formulas (2) over the contact area. Then, integrating the values of the forces with respect to time, we calculated the impact momenta with account for the above-mentioned simplifications by the formula

$$I = \pi R^2 \int_0^{t_c} \left[p_1(r_1)\, r_1^2 + 2\int_{r_1}^{r_2} p_2(r)\, r dr \right] dt\,. \tag{3}$$

The calculations have shown that the momentum of the impact force of an individual drop accounts for less than 10% of the kinetic momentum of the drop at the instant it strikes the surface. This relation can be considered as the mechanical efficiency of a drop in the process under study. Moreover, it has been established that the dependence



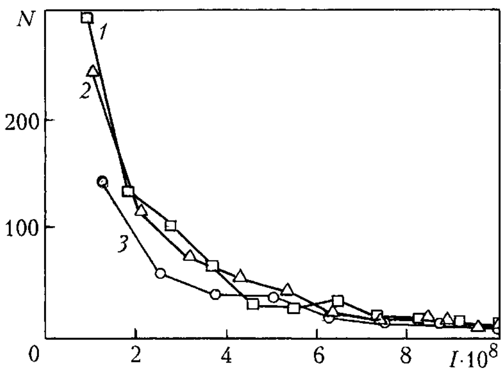


Fig. 4. Distribution of the impact momenta of the drops successively striking the treated surface.

Fig. 5. Distribution of the impact momenta of the drops striking the sites on the jet axis (1) and the sites positioned at a distance equal to 0.25 (2) and 0.5 (3) times its radius.

of the impact force on the time measured from the instant the drop touches the surface to the instant t_c is approximated by a parabola with an error of no more than 5%. The use of this approximation substantially simplifies the calculation of the momentum of the impact force of a large stochastic ensemble of drops.

It has also been established that in the case of a Weibull distribution of the drop diameters, the impact momenta of the drops in a water-drop jet are also distributed by the Weibull law. Despite the introduced limitation on the maximum diameter of the drops, their impact momenta have substantially different values and the variation coefficient of the impact momenta is almost two times larger than the variation coefficient of the drop diameters.

Realizations of the random process $I(t)$ have been obtained by calculating the impact momentum of each drop and the time of its formation. The correlation coefficients of the successive values of the impact momenta do not exceed 0.01, which points to their statistical independence.

The time of travel of a drop τ' from the nozzle to the surface positioned at a distance L from it was determined by integrating the equation of motion, accounting for the air resistance. In the case where the air resistance adheres to the square law, $\tau' = 4\rho_a d(u^{-1} - u_0^{-1})/(3c_z\rho_w)$.

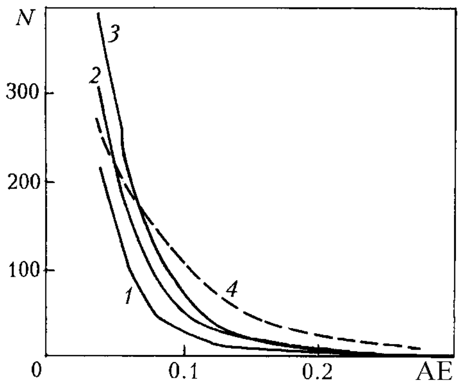
The realizations of the random quantity τ' depend on the diameter and velocity of the drops. Therefore, as a result of the movement of the nozzle relative to the treated surface, the dimensions of the drops reaching a given site of the surface and, accordingly, their impact momenta decrease successively (Fig. 4). This effect is enhanced with increase in the velocity of travel of the nozzle, in the diameter of the water-drop jet, and in the distance between the nozzle and the surface.

In the case of a circular normal distribution of the drop coordinates over the cross section of the jet, the probability that the ith drop will cover the site of the surface with coordinates (x, y) is determined by the relation

$$P_i(x, y) = \frac{A_i'}{2\pi\sigma^2} \exp\left[-\frac{x^2 + y^2}{2\sigma^2}\right]. \tag{4}$$

Since the quantities A_i' are small in comparison with the cross section of the jet, the probability that the drop will strike a given site of the surface is also very small. The time intervals between the successive impacts of drops in the neighborhood of the site with given coordinates is much larger than the mean time of drop formation $\langle\tau\rangle$; therefore, the impact momenta of the drops striking the neighborhood of the given site are statistically independent. Figure 5 shows the distribution of the impact momenta of drops striking different sites of the surface (pressure 100 MPa, nozzle diameter 0.2 mm, velocity of travel 25 mm/sec, jet diameter 4 mm).

The distribution of the distances between the nearest drops acting simultaneously on a surface was investigated in [13]. It has been established that this distribution adheres to the Weibull law. In this case, the mean distances increase and the variation coefficients decrease as the number of nearest drops increases.



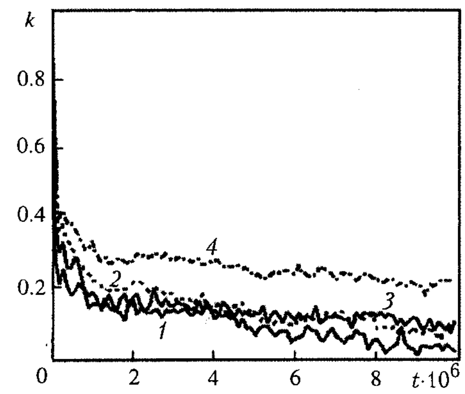
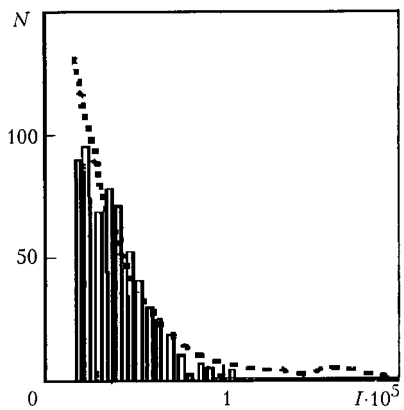


Fig. 6. Distribution of the relative amplitudes of acoustic-emission (AE) signals at various values of the jet parameters p and L: 1) 50 and 50, 2) 100 and 50, 3) 50 and 150, 4) 100 MPa and 150 mm.

Fig. 7. Normalized correlation functions of the acoustic-emission signals at various values of the jet parameters p and L: 1–4) the same values as in Fig. 6.



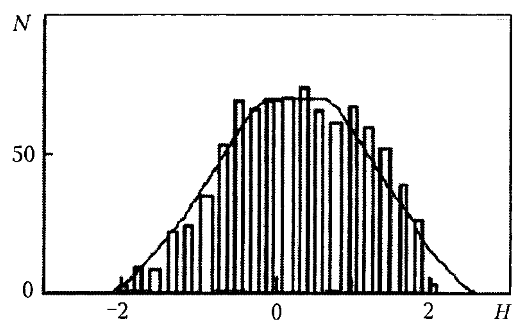


Fig. 8. Experimental (columns) and calculated (solid curve) histograms of the distribution of impact momenta of the drops incident on the sensing element of the detector.

Fig. 9. Experimental (columns) and calculated (dashed curve) distributions of the number of impact momenta of the drops in the jet passing through the sensing element of the detector.

Let us compare the mean distance between the drops acting simultaneously on a surface with the radius of the region of contact of a drop with the surface. The mean distance between two points distributed uniformly on the chord drawn in a random way through the circle of diameter D is equal to $\pi D/12$. Since $D >> \langle d \rangle$, it is evident that the mean distance between the drops is much larger than the maximum radius of the contact site $r_2(t_c)$. Hence it follows that the impact momenta considered as functions of the coordinates are statistically independent.

Experiment and Comparison with the Model. The investigations were carried out at the Laboratory of Water-Drop Technologies of Hannover University. A sample with an 8313 detector of acoustic emission of the Bruel & Kjaer Company (Denmark) with a counting frequency of 700 kHz, fitted to it, was subjected to a water-drop jet flowing out of a nozzle of diameter 0.2 mm. We varied the pressure at the input of the nozzle (50–100 MPa) and the distance from the nozzle to the sample (50–150 mm). The diameter of the jet near the surface of the sample was 2–4 mm. The velocity of travel of the nozzle relative to the sample was 15 m/min. The diameter of the sensing element of the detector, interacting with the jet, was 1 mm.

The detector reacts to the movement of the sample under the action of a water-drop jet. The level of its signal increases when the jet acts directly on the sensing element of the detector. The signals caused by the impacts of drops in the neighborhood of the sensing element were eliminated and the signals whose level accounted for more than 10% of the level maximum for the given experiment were separated and their amplitudes were measured. We constructed histograms of the distribution of the signal amplitudes with respect to the level for different values of the jet parameters p and L (Fig. 6) and compared the correlation functions (Fig. 7) to those calculated by the above-described model.

It should be noted that the experiment gave only a truncated distribution of the signals (see Fig. 6). Moreover, the counting time of the detector (~1.4 μsec) was much larger than the mean time of action of the impact momentum of a drop (~8 nsec) and corresponded to the time of outflow, on the average, of 10 (at a pressure of 50 MPa) or 15 (at a pressure of 100 MPa) drops. Consequently, the signal recorded in the experiment is due to the action of a drop ensemble rather than the action of individual drops. Because of this, the scale of correlation of the detected signals (see Fig. 7) was much larger than the time of action of the impact momenta of individual drops. The calculation of the total momentum of an ensemble of drops striking the surface of the sensing element for the time of signal detection gave distributions of the impact momenta close to the experimental ones (Fig. 8). Estimation of the change in the area of the sensing element covered by the jet and the probability that a drop strikes the sensing element of the detector by formula (4) also led to a satisfactory explanation of the experimentally observed change in the frequencies of the impact momenta of the drops in the jet at the instant it passed through the sensing element of the detector (Fig. 9).

Thus, the statistical model proposed allows one to adequately describe the distribution of the impact momenta of the drops in a water-drop jet acting on a treated surface. The distributions of the impact momenta determined as random functions of the coordinates and the time can be used for estimating the erosion action of a water-drop jet on materials.

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NOTATION

A, constant, MPa; A', area covered by a drop, m^2; AE, relative level of an acoustic-emission signal; B and h, width and thickness of a plane jet, m; c_w, velocity of sound in water, m/sec; c_z, drag coefficient of a drop; d, diameter of a drop, m; d_0, diameter of the nozzle, m; d_s, diameter of the sensing element of the detector, m; D, diameter of a jet with a round cross section, m; H, distance between the jet axis and the axis of the sensing element of the detector, m; I, force momentum, N·sec; k, correlation coefficient; k_u, coefficient accounting for the resistance to the flow in the nozzle; K, kinetic momentum of a drop at the instant it touches the surface, N·sec; L, distance from the nozzle to the treated surface, m; n_c, average number of drops acting simultaneously on the surface; N, number of impact momenta; p, water pressure at the input of the nozzle, Pa; p_w, Zhukovskii impact pressure, Pa; P, probability; Q, rate of water flow through the nozzle, m^3/sec; r, radius of the area of contact of a drop with the surface related to the drop radius; R, radius of a drop, m; t, time, sec; t_c, time of drop spreading, sec; u, velocity of a drop, m/sec; u_0, velocity of the water flowing from the nozzle, m/sec; v, velocity of travel of the nozzle relative to the surface, m/sec; V, volume of a drop, m^3; x, y, coordinates of sites on the surface, m; γ, constant; ν_c, number of drops acting simultaneously on the surface; ρ_w, water density, kg/m^3; ρ_a, air density, kg/m^3; σ, root-mean-square deviation of the drop coordinates from the jet axis, m; τ, time of drop formation, sec; τ', time of movement of a drop from the nozzle to the surface, sec; $\langle ... \rangle$, averaging (mathematical expectation). Subscripts: 0, output of the nozzle; 1, boundary of the central region of contact of a drop with the surface; 2, boundary of the peripheral region of contact of a drop with the surface; a, air; c, contact; i, number of a drop; s, sensing element of the detector; w, water; z, coordinate in the direction of movement of a drop.

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